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A Comprehensive Literature Review: Numerical Solutions for Differential, Integral and Mathematical Equations

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Abstract

In this paper, we systematically review the literature on numerical methods for solving differential equations, integral equations and integro-differential equations. This paper investigates classical numerical techniques, including finite difference methods, finite element methods, spectral methods, collocation methods, and wavelet-based approaches, emphasising the mathematical fundamentals, the convergence behaviour, and computational efficiency. Modern developments, especially on spectral collocation methods, wavelet approaches, fractional calculus techniques and adaptive numeric schemes for boundary value and fractional-order problems are emphasized.

It also explores new machine learning–based methods, especially Physics-Informed Neural Networks (PINNs), Deep Operator Networks, and hybrid computational methods that merge numerical analysis with artificial intelligence. The resulting methods have received much attention since they show better flexibility, faster computation, and more powerful for solving high-dimensional and nonlinear partial differential equations. It also covers recent progress on tensor-network methods and quantum-enhanced PINNs, before moving to require more involved scientific and engineering applications and discussing quantum-enhanced methods based on stochastic differential equation solvers.

This study further discusses the advantages and disadvantages of the different numerical methods based on the accuracy, convergence process, stability, computational complexity, and suggestions for applications to real problems. This review suggests that the computational mathematics of the future is at an exciting juncture as it increasingly shifts towards the use of hybrid and adaptive frameworks combining classical numerical methods with machine learning techniques targeted to efficiently solving large-scale, multi-physics and data-driven problems. Numerical solution of equations arising in science and engineering: Outlook and directions AbstractThis paper provides an informative guide to what is happening now and what is expected to happen in the field of numerical solutions of mathematical equations in the future.

Keywords: Numerical Methods, Differential Equations,Spectral Collocation Methods, Finite Element Method (FEM), Integro-Differential Equations, Physics-Informed Neural Networks (PINNs) ,Fractional Calculus

1. Introduction and Foundation Methods

The various numerical approaches to the solution of differential and integral equations form a foundational pillar of applied mathematics computational science. This discipline covers a wide range of techniques created over the last few decades to solve equations that do not have analytical solutions. Classical approaches are reviewed in [1], but recent work has greatly broadened the methodological toolbox for analyzing capital market outcomes. Classical and upcoming semi-analytical methods dedicated to arbitrary nonlinear diffusion equations, as well as real-world phenomena from a variety of science and engineering fields can be found in [2].

The basic motivation behind numerical methods is that real world problems modeled by differential and integral equations can be very Finite difference and finite element methods, which are traditional methods, have been already made matured through years of development. [3] Finite difference methods (FDM) have been presented to approximate derivatives by using Taylor expansion, while GFEM can be used to tackle boundary value problems offering some alternatives, as shown from the comparative pros and cons of each technique.

2. Spectral and Collocation Methods

Spectral methods have appeared as a powerful substitute to loworder procedures offering exponential convergence rates for smooth solutions. Ref [4] provides the theoretical basis for the observed exponential accuracy of Fourier and Chebyshev differencing techniques by showing that, when approximating an analytic function, errors decay to zero at exponential rates. This result at the core has fueled many developments in spectral-based methods.

An approximate spectral numerical method to solve boundary- and initial value problems based on Chebyshev functions using a trigonometric transformation is presented in [5]. The method offers higher accuracy and computational efficiency in engineering applications. The work [6] compares Legendre and Chebyshev Tau methods for second-order boundary value problem approximations and shows that spectral methods in general (whether using Chebyshev, Legendre or any other orthogonal polynomial method of approximation) are better than the finite element and finite difference methods for problems that require a high degree of accuracy.

The tau method itself, originally introduced in [7], offers a systematic approach to discovering effective numerical solutions of differential equations based on spectral expansions. Then, [8] presents spectral Galerkin and collocation methods using shifted Schröder polynomials for the solution of linear and nonlinear second-order two-point boundary value problems, showing the sprightliness of the spectral techniques.

In particular, collocation methods have undergone more rigorous refinement and development. 9 develop higher-order orthogonal spline collocation methods for periodic boundary value problems with the basis functions built with some care up to the sixth-order approximation. [10] uses spectral collocation methods to boundary control problems for wave equations and shows that convergence is found with non-regularization techniques. What [11] introduces is the extension of spectral element methods to functional differential equations with delayed arguments, yielding convergence of order with polynomial degree.

3. Integro-Differential Equations: Specialized Approaches

Integro-differential equations are a specialized combination of differential and integral operators thus a specialized numerical treatment is necessary. [12] presents a new method of developing Bell wavelet for Fredholm–Volterra integro-differential equations based on the generalized fractional-order Bell wavelets and its collocation techniques. The approach yields exact formulas of Riemann–Liouville fractional integral operators and this provides better accuracy.

A New Chebyshev Collocation Method for Second-Order Fredholm Integro-Differential Equations (2015) [13] is when the Chebyshev collocation method used to solve the second-order as well as high-order Fredholm integro-differential equations with very less computation and approximate solutions with good convergence. This study proposes the Legendre collocation method based on shifted Legendre polynomials for the numerical solution of Volterra and Fredholm IDEs, and the results show a good agreement with their analytical solutions, indicating that the proposed methodology is effective and outperforms the previously developed methods when compared using absolute error measures[14]

Hosseini et al. [15] proposed a quasilinearization-collocation method based on product integration and collocation for nonlinear fractional Volterra integro−differential equations with logarithmic weakly singular kernels. The method applies smoothing methods and establishes error estimates for the new numerical scheme. [16] proposes a quadratic B-spline collocation method to solve the nonlinear integro-fractional differential equations in Caputo sense with the help of Gauss-Legendre quadrature formulas to evaluate the integrals and applies Newton's method to solve the nonlinear systems obtained by collocating the weak form equations.

Another strong pathway is the application of the numerical-analytic method. In [17], Samoilenko's numerical-analytic method is applied to boundary value problems with integral conditions and transformed arguments, building recurrent sequences and proving convergence under standard conditions for this type of method.

4. Finite Element Methods and Modern Refinements

Thus the finite element method (FEM) is still a major player in computational mathematics owing to its versatility and theoretical underpinning. The authors of [18] give a collection of MATLAB functions for spectral collocation methods, offering useful algorithms for Chebyshev, Hermite, Laguerre, Fourier, and sinc interpolation used in eigenvalue, boundary value, and initial value problems.

References [19] defined a functionally connected element method for real finite element based on functional connection theory, which constructed piecewise functions satisfying the intrinsic continuity. FCE alone simulates structures with relative boundary conditions, but it performs worse than traditional element methods, and least squares collocation achieves spectral-like accuracy. They obtain the quasilinearization results for non-linear problems up to the exact solution by using convex Galerkin finite element method based on quintic B-splines to solve the seventh order BVP [20].

Isoparametric approach for geometric accuracy in finite element approximation; curved boundary representation [21]. High-order shape functions do so with respect to complex geometries within an elliptic boundary value problem. Developed in [22] are strongly-form free element methods with auxiliary collocation points that enhance the stability and accuracy of the final discretization without increasing the overall number of nodes in the resulting system.

5. Wavelet-Based Methods and Functional Approaches

Wavelet methods are well suited to the cases in which a localized and adaptive resolution is needed. Hermite wavelets for other applications in areas such as optimal control systems, queuing theory, and medicine have been reviewed in [23], revealing the potential for universal wavelet-based numerical approaches. [24] is a Hermite wavelet based Galerkin method for memory problems of Volterra integro-differential equations, which succeeds in obtaining high accuracy efficiency by wavelet analysis.

In [25], we propose a new approach which is based on hybrid two–dimensional Bernoulli polynomials and modified block–pulse function for system of Fredholm–Volterra integral partial differential equations. The proposed approach offers more accurate accuracy for 2D problems through operational matrices and collocation methods. In [26], tight wavelet frames based on Coiflet wavelet scaling functions framework for fractal-type fractional Riccati differential equations are presented and better than the ones based on Legendre-Galerkin methods and spline-based methods.

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6. Physics-Informed Neural Networks and Machine Learning Approaches

In recent years physics-informed neural networks (PINNs) have arguably provided a paradigm shift in computing methods and server as alternative powerful techniques to solve differential equations. Study [27] elaborates on advanced quasi-Newton optimization algorithms (Self-Scaled BFGS and SSBroyden) for training PINNs and Kolmogorov-Arnold networks with orders-of-magnitude accuracy enhancement on challenging PDEs, inclusive of Burgers, Allen-Cahn, Kuramoto-Sivashinsky, Ginzburg-Landau and Stokes equations.

Binary structured physics-informed neural networks (BsPINNs) [28] decrease the links between two neurons compared to fully connected networks, thus accelerating the convergence and accuracy of equations with quickly variable solutions. The mechanism addresses over-smoothing problems that arise from classical PINNs Reference [29] gives an overview of 40 different machine learning methods for solving PDEs, including comparisons of trade-offs between methods: traditional methods have the advantage of provable guarantees, rigorous error analysis (to a degree), and systematic estimation of the uncertainty, while machine learning methods have the advantages of unprecedented speed and flexibility.

We also explore the concept of Separable DeepONet which has been introduced in [30] to handle the curse of dimensionality in high-dimensional PDE solutions. The factorization method decreases the numerical cost linearly with the density of discretization, resulting in comparable or even superior accuracy to the standard DeepONet at greatly minimized training times. [31] introduces hard constrained sequential PINNs (HCS-PINN), where temporal continuity of the solution in neighbouring time segments is guaranteed through solution ansatz for continuous time problems, and show improved convergence relative to existing time-dependent PINNs by not requiring continuity loss terms.

Additional details: PINNs seems to be enhanced by coarse mesh finite element pretraining which yields a robust initialization that guides the rapid satisfaction of physical laws while reducing the computational cost [32]. AL-PKANs : [33] proposals augmented Lagrangian physics-informed Kolmogorov-Arnold networks (AL-PKAN) which avoid spectral bias in multilayer perceptrons and adaptively balance penalty factors during optimization with learnable Lagrangian multipliers.

Other works include the deep operator networks and their related objects in the broader machine learning ecosystem for PDEs. In [34], the authors review a wide class of neural networks methods for PDEs, including PINNs, deep BSDE methods, operator learning methods, and establish mathematical foundations for such emerging techniques.

7. Fractional and Advanced Calculus Methods

The role of fractional calculus as a mathematical tool for semiempirical modeling of memory effects and anomalous dynamics in complex systems is rapidly growing. (2001) Typeset by Foil, the fractional q-integro-differential equations: Numerical Methods, and Fixed Point Theory 18 of 8783Study 6 fractional q-integro-differential equations: Schauder and Banach fixed point theorems 7 Develop numerical methods that are finite difference and trapezoidal methods. Ref. [36] described numerical methods based on Vieta-Fibonacci polynomials for fractal-fractional pantograph differential equations and systems with the help of Caputo, Atangana-Baleanu and Caputo-Fabrizio integrals.

Recently, [37] has proposed two collocation methods for Caputo-type non-local boundary value problems based on linear fractional differential equations. Using spline collocation on graded grids, we obtain optimal global convergence bounds and super-convergence results under further assumptions. They do this in [38], where they derive spectral shifted Legendre Gauss-Lobatto collocation methods for two-dimensional initial-boundary fractional diffusion equations with exponential convergence rates, and use implicit Runge-Kutta methods to solve the resulting systems.

Reference [39] introduces a Genocchi wavelet collocation method to solve systems of ordinary differential equations derived from fractional Polio models, which transforms the problem into nonlinear algebraic equations through operational matrices and points of collocation. [40] uses Lucas wavelet methods in cardiac rhythm modeling of nonlinear and fractional Van der Pol oscillators, showing a much better performance to Runge-Kutta fourth order methods.

Computational Efficiency and Convergence Analysis

The properties of convergence of numerical schemes are still at the core of choice and validation of the method. [41] makes equivalences between nonlinear higher-order fractional differential equations FDE and integral equations, show that the continuity assumptions imposed in the literature are not enough for any Caputo FDEs and derive sufficient conditions that the solutions of an integral equation are also solutions of an FDE (and conversely). • [42] develops error-controlled numerical methods for linear ordinary differential equations based on the use of Chebyshev spectral methods, augmented by a posteriori quasi-Newton validation, via theoretical properties of corresponding operators as compactness and invertibility.

The well-posedness of a fractional Zakharov-Kuznetsov equation (ZKE) was discussed in [43] where a fully discrete Fourier spectral Galerkin method was developed, with uniform convergence, and the spectral convergence order (where is the order of smoothness of the solutions) with exponential convergence for analytic solutions. The stiff fractional terms are treated via an integrating-factor Runge-Kutta time discretization. [44] studied the exponential Runge-Kutta Galerkin methods for reaction-diffusion systems with nonsmooth initial data in a fractional Sobolev space and giving sharp fully discrete error estimates.

Real-world applications are improved through error analysis and adaptive methods. In [45], an adaptive coordinate-stretched spectral collocation method along with the quasilinearization method are developed for singularly perturbed boundary value problems. The proposed method adaptively determines the stretching parameters using the residual based spectral criteria, enabling near-head accuracy without any manual parameter tuning. Due to the inherent high accuracy of spectral methods, the authors in [46] applied the spectral-grid method to two-parameter singularly perturbed differential equations with a grid that adapts to the boundary layer structure.

9. Specialized Applications and Domain-Specific Methods

General numerical methods motivate specialization given application diversity. That is a very natural model of physical sustainability, [47] however shows stability and efficiency of Chebyshev polynomial methods for second-order Fredholm integro-differential equations. [17] focuses on particular numerical-analytic methods for the case of systems with transformed arguments and integral boundary conditions, with examples that show practical realizations of the method.

[48] propose to use mixed formulation based PINNs in inverse and data-driven problems combining finite element methods concepts that can be used fully coupled in solving heterogeneous elasticity and diffusion problems while remaining compatible to traditional FEM. This partially relates with reducing the cost of training of neural networks with regard to its accuracy highly observable in the comparison of finite difference-PINNs (FD-PINNs) with AD-PINNs done in [49] showing competitive performances in simple scenarios of steady incompressible flow.

Similarly, one needs to treat stochastic systems in their own unique way. Quoc Ninh Trinh, and Chang-hoon Lee, The Milstein method for multidimensional multiscale stochastic differential equations, Numer. Math. This method computesG multiple stochastic integrals and we propose novel strong convergence order theorems without analytical solutions. [51] constructs a dual-fractional framework to a class of stochastic differential equations, where conformable fractional calculus is applied to both the system dynamics and the driving noise.

10. Emerging Trends and Future Directions

Current studies show that hybrid techniques with the combination of one or more techniques are more common. This approach, which is based on [52], presents kernel-free boundary integral methods combined with deep learning operator networks for solving boundary integral equations stemming from elliptic PDEs, while still retaining second-order accuracy. This is an excerpt from a cover article, published in [53], on tensor train (TT) decompositions in compression of terabyte-scale operators into kilobytes using tensor network space-time spectral collocation methods in exponential convergence and applications with up to tens of thousands of speed-up in numerical simulation.

Adaptive and multi-method frameworks are growing necessities in analysing multi-physics and multi-scale problems. [54] generalizes tensor network approaches to non-linear time-dependent convection-diffusion equations, with the "Step Truncation TT-Newton" formulation to track tensor train rank in the course of non-linear iterations. The work in [55] proposes a new paradigm of quantum physics-informed neural networks (QPINN) using trainable embedding with the promise of bringing together the best of quantum and classical computing for nonlinear PDEs.

The field is maturing towards an ecosystem of practical software. (NEK)

Conclusion

The development of closer numerical solution of differential, integral and mathematical equations shifted from classical methods toward an advanced and more sophisticated scenery of methods, such as spectral methods, advanced finite element methods, methods based on neural nets, and hybrid frameworks. Each method has its own strengths: spectral methods lead to exponential convergence for smooth problems, finite element methods expose geometric flexibility, and machine learning based strategies facilitate computational efficiency and data integration. It is becoming more common practice that complementary strengths are combined — fast approximations produced on the basis of physics and neural nets can then be refined with traditional numerical methods, or neural network-based acceleration is added to classical framework. The evolution of this field will likely focus on developing adaptive techniques, quantifying uncertainty, and integrating classical and learning-based methodologies to deal with the increasing sophistication of real-world applications from the sciences and engineering.

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